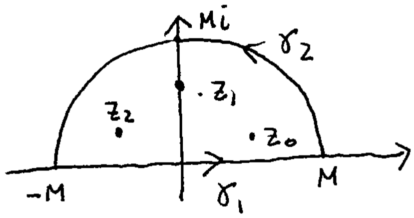
26(a) $I = \int_0^{+\infty} \frac{dx}{a x^6 + b} = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{dx}{a x^6 + b}$, $a, b > 0$.



$f(z) = \frac{1}{a z^6 + b}$ pols? $a z^6 + b = 0 \Leftrightarrow z^6 = -\frac{b}{a} = \frac{b}{a} e^{i\pi}$

$z_k = \left(\frac{b}{a}\right)^{1/6} e^{i(\pi + 2\pi k)/6}$, $k = 0,1,2,3,4,5$

$z_0 = \left(\frac{b}{a}\right)^{1/6} e^{i\pi/6}$, $z_1 = \left(\frac{b}{a}\right)^{1/6} e^{i\pi/2}$, $z_2 = \left(\frac{b}{a}\right)^{1/6} e^{5i\pi/6}$

$\gamma = \gamma_M = \gamma_1 + \gamma_2$, $\int_\gamma f(z)\,dz = \int_{\gamma_1} f + \int_{\gamma_2} f = 2\pi i \operatorname{Res}(f, z_0) + 2\pi i \operatorname{Res}(f, z_1) + 2\pi i \operatorname{Res}(f, z_2)$

$$\operatorname{Res}(f, z_k) \underset{\text{pol simple}}{=} \lim_{z \to z_k} (z - z_k) f(z) = \frac{1}{6a z_k^5} = \frac{1}{6a (b/a)^{5/6}} e^{-5i(\pi + 2\pi k)/6}$$

Sumem els residus:

$$\operatorname{Res}(f, z_0) + \operatorname{Res}(f, z_1) + \operatorname{Res}(f, z_2) = \frac{1}{6a (b/a)^{5/6}} \left(e^{-5i\pi/6} + e^{-5i\pi/2} + e^{-25i\pi/6} \right) =$$

$e^{-25i\pi/6} = e^{-i\pi/6}$

$e^{-5i\pi/2} = e^{-i\pi/2}$

$$= \frac{1}{6a (b/a)^{5/6}} \left[\cos\left(\frac{5\pi}{6}\right) + \cos\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{6}\right) - i \left(\sin\left(\frac{5\pi}{6}\right) + \sin\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{6}\right) \right) \right] =$$

$\cos\left(\frac{5\pi}{6}\right) + \cos\left(\frac{\pi}{6}\right) = 0$ $\sin\left(\frac{5\pi}{6}\right) + \sin\left(\frac{\pi}{6}\right) = 2 \sin\left(\frac{\pi}{6}\right) = 2 \cdot \frac{1}{2} = 1$

$$= \frac{-2i}{6a (b/a)^{5/6}}$$

$$\int_{\gamma_1} f(z)\,dz = \int_{-M}^{M} \frac{dt}{a t^6 + b} \xrightarrow[M \to +\infty]{} 2I$$

$\gamma_1(t) = t$, $t \in [-M, M]$, $\gamma_1'(t) = 1$

$$\int_{\gamma_2} f(z)\,dz = \int_0^\pi \frac{M i e^{it}}{a M^6 e^{6it} + b}\,dt = \int_0^\pi \square_M\,dt \xrightarrow[M \to +\infty]{} 0$$

$\gamma_2(t) = M e^{it}$, $t \in [0, \pi]$, $\gamma_2'(t) = M i e^{it}$

$|\square_M| \le \frac{M}{a M^6 - b} \xrightarrow[M \to +\infty]{} 0$ uniformement $\forall t \in [0, \pi]$

Així, si fem $M \to +\infty$: $2I = 2\pi i \frac{(-2i)}{6a (b/a)^{5/6}} \Rightarrow I = \frac{2\pi}{3a} \left(\frac{a}{b}\right)^{5/6}$