Lab session

1. The version of Jukes-Cantor model for 2 states is known as CFN model (due to Cavender-Farris-Neyman). Consider the claw tree on 3 leaves and write in SINGULAR the parameterization of the CFN model as a ring map $\mathbb{R}[p_{000}, \ldots, p_{111}] \to \mathbb{R}[\text{parameters}]$.

   (i) What is the dimension of the corresponding variety?
   (ii) What is the ideal of the corresponding variety?

Note: use always the homogeneous parameterization.

2. Repeat the previous exercise considering now the Jukes-Cantor model.

3. On the following tree on 4 leaves,

   ![Tree](image)

   consider the CFN model (see exercise 1). Write the parametrization of this model on this tree in SINGULAR.

   (i) What is the dimension of the corresponding variety?
   (ii) What is the ideal of the corresponding variety?
   (iii) Are the linear generators (or some of them) phylogenetic invariants?
   (iv) Is the tree topology generically identifiable?
   (v) Are the parameters of the model generically identifiable?

4. On the same tree as in the previous exercise, consider a General Markov model on 2 states $\{0, 1\}$. Write the parameterization in SINGULAR.

   (i) Check that the $3 \times 3$ minors of the flattening of $(p_{000}, \ldots, p_{111})$ along the bipartition $12 | 34$ vanish on the corresponding algebraic variety.
   (ii) Is the tree topology generically identifiable? (do not attempt to compute the phylogenetic ideal)
   (iii) Are the parameters generically identifiable? (do not attempt to compute the phylogenetic ideal)
   (iv) Could the corresponding phylogenetic ideal be generated by the $3 \times 3$ minors above? (do not attempt to compute the phylogenetic ideal)

5. On the same 4-leaves tree as above, consider a General Markov Model and write the corresponding parameterization in SINGULAR.

   (i) Check that the $5 \times 5$ minors of the flattening of $(p_{AAAA}, \ldots, p_{TTTT})$ along the bipartition $12 | 34$ vanish on the phylogenetic variety.
(ii) Check that they are *phylogenetic* invariants.

(iii) What is the dimension of the variety defined by these minors? (guess before computing it in SINGULAR).

(iv) Is the ideal of the phylogenetic variety generated by these minors? (do not try to compute the whole ideal of the variety!)