Detection of Current Distribution in Bulk Samples With Artificial Defects From Inversion of Hall Magnetic Maps

M. Carrera, X. Granados, J. Amorós, R. Maynou, T. Puig, and X. Obradors

***Abstract*—A two-sided inverse Biot-Savart algorithm for the computation of critical current distributions in High Temperature Superconductor (HTS) pellets is introduced. It is based on simultaneous measurement of the magnetic field on top and bottom of the bulk sample with a double Hall probe, and the authors' discretization and QR inversion scheme. It yields current maps which are detailed in the ab-plane for each half of the bulk sample along the c-axis. The computed current is a O(1/z)-weighted average of the real current along the c-axis over each slice. This is the best possible resolution for current maps along the c-axis, regardless of inversion scheme.**

We demonstrate the new procedure by analyzing a non-c-homogeneous simulation and a real HTS pellet of cylindrical geometry with asymmetrically drilled holes on both faces.

***Index Terms*—Critical current maps, hall magnetometry, HTS bulk samples, inverse Biot-Savart problem.**

I. Introduction

THE application of YBCO melt-textured blocks to engineering problems requires an understanding of the current patterns in them, as well as nondestructive quality control procedures. This leads to the inverse Biot-Savart problem, solvable on 2-dimensional domains (see [1]–[4]), and unsolvable for general 3-dimensional current patterns.

This note discusses the authors' work on the inverse Biot-Savart problem on planar-crystallized HTS bulk samples. We show how our 2-dimensional discretization and QR-inversion scheme of [4]–[6], when applied to such bulk samples, yields the average of the real current J along the c-axis weighted by a function O(1/z). This procedure is refined into a two-sided inverse Biot-Savart algorithm, in which simultaneous measurements of B_z above and below the sample are taken, the sample is subdivided in two vertical slices, and the c-average of the current **J** is obtained for each slice.

It follows from previous work by Eisterer [7] and the authors [5] that these c-averages and vertical resolution are the maximal information that inversion of the Biot-Savart problem by any procedure may yield.

To illustrate this discussion, we present: a simulation with a current density varying along the c-axis; and a real bulk sample which is a cylinder into which holes have been drilled. These holes are not homogeneous along the c-axis, yet our two-sided inverse Biot-Savart procedure finds their influence in the circulating currents.

II. Feasibility of Current Computations in HTS Bulk Samples

The Biot-Savart operator yielding the magnetic field B induced by a current density **J** on the sample is invertible in 2-dimensional domains, and becomes invertible in bulk samples for currents that are:

(a) planar (i.e., $\mathbf{J} = (J_x, J_y, 0)$);

(b) homogeneous along the c-axis (i.e., $\mathbf{J}(x, y, z) = \mathbf{J}(x, y)$).

The authors have adapted in [4], [6] the discretization procedure of [2] to both HTS thin films and bulk samples, with the above hypotheses (a) and (b) as the only regularity assumptions made on the critical current in the sample, and are able to obtain from measurements of the vertical magnetic field B_z above the sample a 2-dimensional map of the critical current $\underline{J}$, with a resolution comparable to that of the magnetic measurement, detecting irregularities such as multiple domains or welds [8], [9]. The discretized Biot-Savart problem is solved by an over-determined QR inversion to achieve results that are robust in front of measurement errors. An implementation of this algorithm is offered for free use at the web http://www.jaumetor.upc.es/caragol.

While the above planarity hypothesis (a) holds for any planar-crystallized sample, it is known that hypothesis (b) does not hold for most bulk samples, i.e. their critical current **J** has inhomogeneities along the c-axis. Thus two questions arise naturally:

Question 1

What is the meaning of the current $\underline{J}$ obtained by solving the inverse Biot-Savart problem, imposing the c-homogeneity hypothesis $\mathbf{J}(x, y, z) = \underline{J}(x, y)$?

Question 2

Is it possible to solve the inverse Biot-Savart problem with only the planarity hypothesis (a) $\mathbf{J} = (J_x, J_y, 0)$, and obtain 3-dimensional critical current maps?

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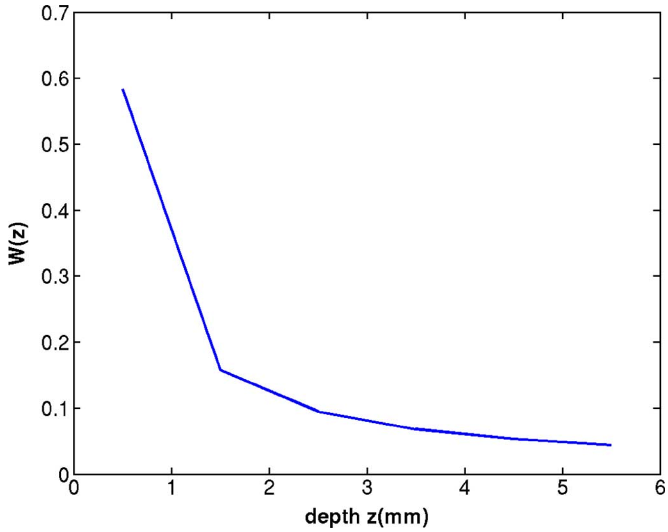

Fig. 1. Share of every 1 mm c-slice in a 6 mm-thick sample towards the (1/z)-weighted average of the current J that our inverse Biot-Savart algorithms retrieve.

Answer 1

The answer to question 1 should arise from applying the Biot-Savart law to an unknown planar current $\mathbf{J}(\mathrm{x,y,z}) = (\mathrm{J_x, J_y}, 0)$ on the bulk sample, also to an unknown planar and c-homogeneous current $\underline{\mathrm{J}}(\mathrm{x,y})$, and imposing that both currents generate the same magnetic field $\mathrm{B_z}$ on the points where it has been measured. The current **J** may be replaced by the magnetization M in schemes, such as the authors, that rely on finding M and then **J** as curl M. The resulting relation between the real M and the c-homogeneous version $\underline{\mathrm{M}}$ is complex, and depends on the position (x,y) in the ab-plane. Nevertheless, simplifications on the general expression, and heuristic evidence from simulations run by the authors suggest that the c-homogenous $\underline{\mathrm{M}}$ and $\underline{\mathrm{J}}$ computed from measurements of $\mathrm{B_z}$ above the sample, with our QR inversion procedure for the Biot-Savart problem are weighted averages of the real magnetization M and current **J** along the c-axis, i.e.

$$\underline{J}(x,y) = \int J(x,y,z)W(z)dz \tag{1}$$

where the weight function W(z) is approximately $\mathrm{O}(1/|\mathrm{z}|)$, and $\mathrm{z} = 0$ is the plane where $\mathrm{B_z}$ has been measured. As Fig. 1 illustrates for the typical dimensions of HTS bulks, this weighting is not so top heavy that a fine superficial layer obscures the current circulation below it.

Simulation 2 from [5] provides a typical check of this formula: it consists of a virtual c-stack of 12 1-mm thick cylinders, each of them with a regular current with constant density, but this density varies along the c-stack. The vertical magnetic field $\mathrm{B_z}$ that such a stack generates at a height of 0.1 mm above and below the sample was computed, and our two-sided inverse Biot-Savart computation was performed, subdividing the sample into two c-stacked cylinders of thickness 6 mm, and each cylinder into a rectangular grid of elements. As Table I shows, the result of the computation agrees with the (1/z)-average of the real density of formula (1), with a margin of error of 2%.

TABLE I
(1/z)-WEIGHTED AVERAGE OF $\mathrm{J_v(z)}$

Position	WEIGHTED AVERAGE	Computed current
Top half	9.02 x 10^7	8.9 x 10^7
Bottom half	5.98 x 10^7	6.1 x 10^7

(1/z)-weighted average of $\mathbf{J}_\mathrm{v}(\mathrm{z})$ vs c-homogeneous $\underline{\mathrm{J}}_\mathrm{v}$ yielded by 2-sided inverse Biot-Savart computation in sample S1 from [5].

Further checks on virtual samples with homogeneous current density $\mathrm{J_v(z)}$ along every ab-plane, and varying thickness, radius and density distribution along the c-axis have yielded computed currents J which differ in at most 25% from the (1/z)-average of the originally imposed current J.

Answer 2

It is not possible to compute a 3-dimensional critical current map for an ab-oriented crystallized bulk sample from measurements of the magnetic field B around it. This answer was pointed out by Eisterer [7], and is documented by simulation 1 from [5], in which a current density averaging $10^8\ \mathrm{A/m^2}$ is distributed in a cylinder so as to yield a maximum magnetic field of 5×10^{-5} T at distances of 0.2 mm or further.

The finest possible computation of J in a bulk sample is obtained by subdividing it in 2 slices of comparable thickness along the c-axis, and seeking separate maps for each slice. This yields a current $\underline{\mathrm{J}}$, weighted average of the real current **J** along the c-axis over the slice as in formula (1). We will refer to the procedure employed to do this as the two-sided inverse Biot-Savart algorithm. It is a refinement of the inversion procedure for thin films and bulks described by the authors in [4].

The input for the computation is the measure of the vertical magnetic field $\mathrm{B_z}$ on grids above and below the sample. The sample (or a box containing it) is then subdivided into two slices of similar thickness along the c-axis (the top and bottom slices), and each slice into a rectangular grid of elements. The magnetization M is assumed to be constant in each element, thus the measured magnetic fields become linear functions of the unknown values of M, which can be obtained by solving the resulting linear system [4]. Let us remark that the resulting system of equations is coupled, i.e. the equation for the magnetic field at each measurement point includes the terms provided by all discretization elements in both the top and bottom slices of the sample.

To achieve separate current maps for the two slices of the sample, it is essential that $\mathrm{B_z}$ be measured on both sides of the sample. If it is measured on only one side, the shielding effect of simulation 1 in [5] becomes possible, and the existence of large c-inhomogeneous perturbations of **J** affecting only slightly the magnetic field at the points of measure means that the inverse Biot-Savart problem is too ill-conditioned to solve. Even if we use all space components of the magnetic field, above the sample and on its sides, the inverse Biot-Savart problem is so ill-posed (condition numbers starting at 10^3) that any noise in the measurement of B prevents the inversion.

On the other hand, if one measures the vertical magnetic field $\mathrm{B_z}$ on grids above and below the sample, and uses the above two-sided inverse Biot-Savart discretization procedure with a resolution that is 2–3 times the resolution used in the measurement of $\mathrm{B_z}$, the resulting discretized problem may be inverted,

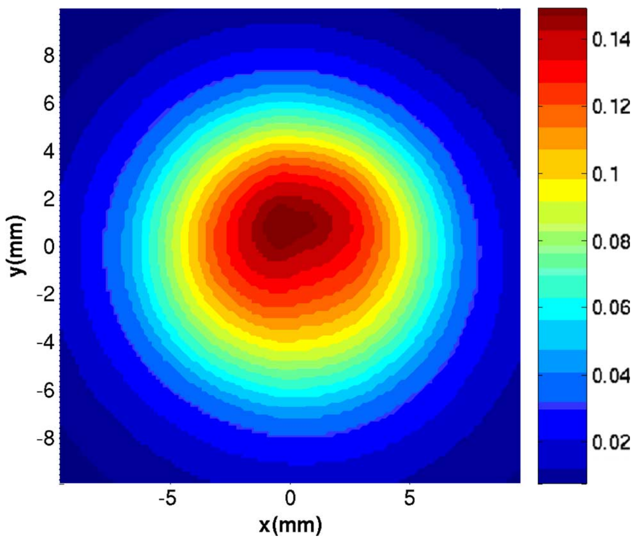

Fig. 2. Map of magnetic field Bz measured in top face of sample.

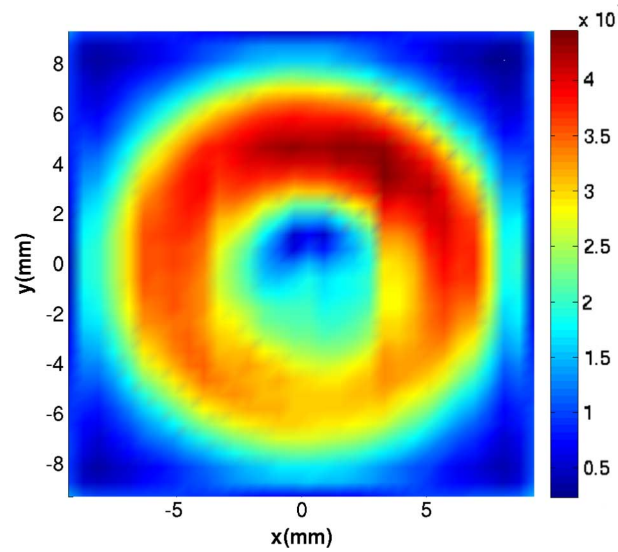

Fig. 3. Computed map of modulus of the current density, J, in top slice of sample.

and has a condition number low enough for the results to be meaningful, as the simulations 2, 3 from [5], and the real sample of Section III illustrate.

III. YBCO Samples With Artificial Defects

Single domain melt-textured YBCO samples were grown by top seeding growth method. Two cylinders (A and B) of equal geometry (diameter 15 mm, height 6 mm) were extracted from those samples. A welded cylinder (A top + B bottom) was built. In this, a hole was drilled with diameter 3.8 mm, depth 4 mm, centered at the midpoint of a radius in the top face (hole A); a second hole was drilled with diameter 2 mm, depth 4 mm, in the bottom face (hole B).

Hall probe measurements of the remanent magnetic flux maps $\mathrm{B_z(x,y)}$ at 77 K have been made after a FC process under an applied field of 1 T, perpendicular to the sample surface (ab-plane). An electromagnet was used in the magnetization process to guarantee homogeneous magnetic field region over all the sample. The probe is scanned at a flying distance of $\approx 100\ \mu\mathrm{m}$ in steps of 160 $\mu\mathrm{m}$, and for each sample both the top and bottom sides of the composite cylinder are probed using an improved 2-Hall probe system that allows us to obtain a correct correlation between the positions of both sides in the maps.

Hall probe measurements of $\mathrm{B_z(x,y)}$ on two squares of $30 \times 30\ \mathrm{mm}^2$ were made, and $19.2 \times 19.84\ \mathrm{mm}^2$ windows covering the top and bottom of the composite cylinder were used to apply the two-sided inverse Biot-Savart algorithm, computing two critical current maps corresponding to a subdivision of the sample in two slices along the c-axis. The two slices had equal thickness 6 mm. Both the top and bottom slices were subdivided in $0.58 \times 0.58\ \mathrm{mm}^2$ rectangular grids, on which the magnetization M and the current $\underline{\mathrm{J}}$ were computed. The coarse resolution in the ab-plane was selected because it made the resulting linear systems 13:1 overdetermined, with condition number 2.50.

Figs. 2 and 4 display the level curves of the magnetic field measured on the top and bottom faces of the sample, showing the effect of the drilled holes on the distribution of the trapped field. Prior to drilling any hole, the bottom cylinder had a slightly higher field trapping capacity than the top one [5]. The sample has peak magnetic field values of 0.15 T (top) and 0.18 T (bottom), and the holes cause a displacement to the opposite side of the peak point.

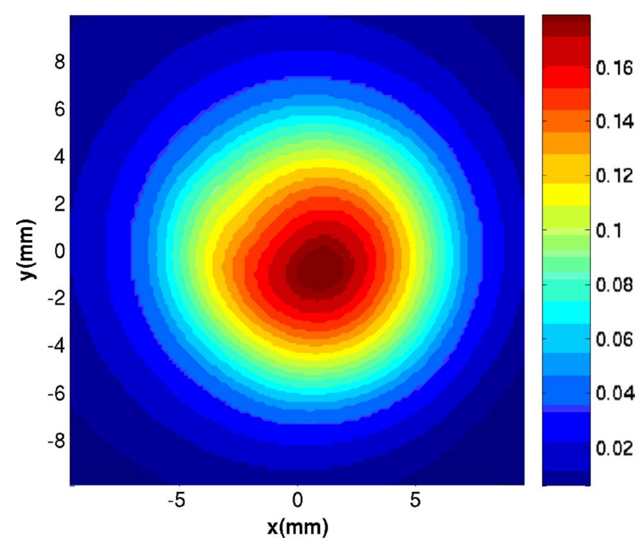

Fig. 4. Map of magnetic field Bz measured in bottom face of sample.

Figs. 3 and 5 show the results of the computation of the critical current from the measurements of the magnetic field with our two-sided inverse Biot-Savart algorithm. The computation of the critical current in the sample before drilling any hole was performed in [5], and showed that it was concentrated in a ring around the edge of the cylinder. The drilled hole results in a narrowing and distortion of the current ring in the top slice, with a maximum density of $3.5 \times 10^7\ \mathrm{A/m}^2$ (Fig. 3). The bottom hole results only in a distortion of the current ring in the bottom half (Fig. 5).

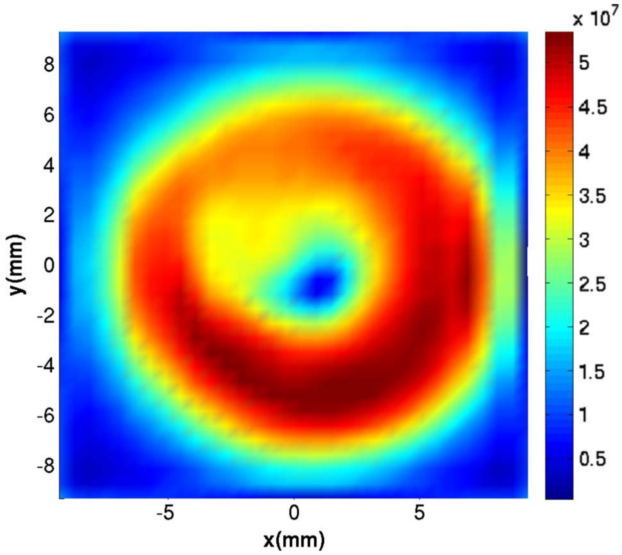

Fig. 5. Computed map of the modulus of the current density, J, in bottom slice of sample.

IV. Conclusion

Our two-sided inverse Biot-Savart algorithm is able to compute separate ab-maps of the critical current J in two slices stacked along the c-axis of a crystallized HTS bulk sample.

For each slice, the yielded current $\underline{J}$ is the average of the real current along the c-axis with a weight O(1/z), where $z = 0$ is the surface of measure of B_z.

This procedure yields current maps with a fine resolution along the ab-plane for samples with any geometry, and is capable of detecting irregularities such as holes, even if they are not homogeneous along the c-axis.

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